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AN ANALYTICAL APPROACH TO THE THEORY OF STRUCTURAL IDENTIFICATION

We further develop the elements of a qualitative theory of structural identification of linear dynamic systems through an analytical solution of the abstract realization problem (in the sense of Erugin-Kalman-Mesarovic) in a class of strong differential (A,B)-models. The theoretical apparatus is constructed on the basis of introducing a special operator whose structure constitutes a modification to the canonical Rayleigh-Ritz ratio.

Keywords: *identification, realization theory, strong irrefutable (A, B)-model, Rayleigh-Ritz operator, structural optimization.*

1. INTRODUCTION

This paper is a continuation of the study initiated by these authors in a series of articles (Daneev, 1994a,b), and is ideologically related to publications (Daneev, 1995, 1997, 1999a-c, 2000; Erugin, 1952; Lakeyev, 1998; Rusanov, 1999; Van der Schaft, 1987; Vassilyev, 1996; Willems, 1979). The phrase *"Analytical approach..."* that appears in the title emerged in the cited references for some fundamental reasons. One implies that the whole of the conceptual construction below is made in an abstract (theoretical-system) presentation. The other reason is due to the fact that the theoretical-model approach suggested below "differentiates" the problem of structural identification into five research areas which axe put in order (between them) by a deductive process of "decision making". The areas themselves, in accordance with the indicated order, may be terminologically designated as: 1 - *"analysis of types"*, 2 - *"synthesis of types"*, 3 - *"analysis of representations"*, 4 - *"synthesis of representations"*, and 5 - *"construction"*. These terms are treated (and used in follows) as implying the following:

- *"analysis of types"* must establish, in terms of mathematical definitions of key criteria universes, the type [1] of structure, and (in this paper, according to standard "discrete-continuous" and "linear-nonlinear" dichotomies) by an immanent dynamic system that "services" the observations imposed *(a posteriori* information) in the form of *"trajectory, control"* pairs (see Theorem 1 below);
- *"synthesis of types"* should identify the conditions of the previously established type of structure of the modeled dynamic object when a posteriori information is expanded (Theorem 2);
- *"analysis of representations"* serves to determine the kind[2] of structure, or, more precisely, its analytical representation, in the position when its type is known (identified);
- *"synthesis of representations"* is intended to describe the invariance of the type of structure identified with respect to the expansion of a posteriori information;
- *"construction"* implies manipulation (ideally, optimization) of numerical values of free model parameters within the framework of the structure which is fixed as a result of solving the four preceding problems.

It is reasonable to expect that the elements (and primarily "hypostasizing") of deductive theoretical-model analysis, as suggested in this paper, will permit us to develop an elegant, adequate structural theory encompassing the above- mentioned five areas.

2. KEY CONCEPTS

Structural considerations play a crucial role in both the analysis and synthesis of systems of quite different types (Ginsberg, 1998; Vassilyev, 1996). Therefore, the starting point for our investigation will be the concept of the dynamic system (D-system) that is formalized in theoretical- multivariate terms (see Definition 1 below). At this level, axiomatization of the behaviour of the D-system is postulated in terms of the theory of sets as a ratio (Defintion 1).

[1] In N.Bourbaki's interpretation the type of structure (the terminology used in this paper) is fitted by its genus (Bourbaki, 1965).

[2] The term "kind of structure" must be formally perceived as a narrowing of the type of structure to a fixed class of mathematical models with a finite identification dimensional representation (Daneev, 1994a).

Furthermore, although many of the concepts of structural identification may be defined on the basis of nothing but theoretical-multivariate constructions of a D-system and a formal identification process (Definition 2 (Daneev, 1994a)), it becomes possible to obtain substantive mathematical results only by introducing additional structures. In this context, the approach used later in the text to construct the elements of general theory of structural identification of D-systems is epistemologically as follows:

- the basic concepts of structural identification are introduced through a formalization (i.e. based on a verbal description of some intuitive representation, an accurate mathematical definition of this concept is

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given using, for this purpose, a minimum mathematical structure allowing for its correct interpretation) [3]; and

- then on the basis of the basic concepts obtained as a result of formalization, a consistent advance of the theory is accomplished by adding new mathematical structures needed to investigate the various identificability properties of the structure of the D-system (such a procedure makes it possible to elucidate how actually fundamental some concrete structural property is, as well as what minimum set of assumptions is needed for the identification process to reveal this property or, for a given relationship, to be satisfied for it).

After this attempt to give an account of the motivations for this study, we now present the definitions of the basic mathematical constructions of D-systems; the term *structure* will be understood throughout this paper in the sense of Bourbaki, and the representation (Δ, S_Δ) - will imply that the set Δ is endowed with the structure S_Δ.

Definition 1 (Willems, 1979). *The D-system is said to be an ordered set of three pairs* $\Sigma = \{(T,S_m),(W,S_W),(\Omega,S_\Omega)\}$, *in which T is a (abstract) set of times, W is the alphabet of signals, and Ω is the behaviour of the D-system* Σ *(family of its trajectories* $\omega: T\to W$*), and the fixed procession* (S_T, S_w, S_Ω) *is said to be the structure of the D-system Σ.*

3 Thus, for example, in spite of the wide diversity of structures of mathemnatical analysis, it turns out that they are created of the simplest structures of the following types: order-type (reflecting the comparison), topological (giving the notion of proximity), and algebraic (determining the combination of the addition and multiplication operations).

Below, with reference to the structure S_T, we shall confine our consideration to a linear ordering of the set T by the quotient < (Mesarovic, 1975). With such a setting, any pair (T_*,T^*) of subsets of the set T, that $T_* \cup T^* = T$, $T_* \neq 0 \neq T^*$ and from $t_* \in T^*$, $t^* \in T^*$ it follows that $t^* < t^*$, is called (Engelking, 1985) the section of the set T. Furthermore, it is implied that (T_*,T^*) is a jump if ($\exists$ sup $T_* \in T^*$) (((inf T* (T* and (T*,T*) is a gap if ((((sup T* (T*) (((inf T* (T*)) (Engelking, 1985).

Definition 2. *The D-system* $\{(T, S_m), (W,S_w),(\Omega,S_\Omega)\}$ *is of discrete type if the structure* S_m *is such that any section in T is a jump and, accordingly, of continuous type if none of the sections of the set T is a jump or a gap.*

Let W be the Abelian group with addition as a group operation $\oplus$ and let R be a certain field. Assume further that the mapping $\otimes$ is specified: $\otimes: R \times W \to W$, such that the set of four $(W, \oplus, R, \otimes)$ is the vector space over the field R (representation of the structure S_w).

Definition 3 (Mesarovic, 1975). *The D-system* $\{(T, S_T), \{W, S_W), (\Omega,S_\Omega)\}$ *is of linear type if the structure* S_Ω *is such that*

$$\omega, \omega' \in \Omega \Rightarrow \exists \omega^* \in \Omega :$$

((t (T) (*(t) = ((t) ((’(t) (W;

$$((\Omega \,\&\, \alpha \in R \Rightarrow \exists \omega^* \in \Omega :$$

$$(\forall t \in T)\ \omega^*(t) = \alpha \otimes \omega(t) \in W.$$

Note (and this is very important for our subsequent discussion) that each nonsingular subset $\Omega^\#$ of the behaviour $\Omega \subset W^T$ induces (Mesarovic, 1975) a subsystem $\{(T, S_T), (W, S_w), (\Omega^\#,S_\Omega{}^\#)\}$ of the D-system $\Sigma = \{(T, S_T), (W, S_w), (\Omega,S_\Omega)\}$. In this statement, $\Omega^\#$ is said to be the partial behaviour of the *D*-system Σ, and because the process of structural identification models the structure of the *D*-system, with the only purpose of extending the stylistic possibilities, we shall write on frequent occasions (in the context of an a posteriori modeling) the phrase *"set of observations"* instead of the phrase *"partiual behaviour of the D-system”*. We must not be discouraged by the fact that in this treatment the linearity of the *D*-system $\{(T, S_T), (W, S_w), (\Omega,S_\Omega)\}$. guarantees no linearity of its arbitrary subsystem $\{(T, S_T), (W, S_w), (\Omega^\#,S_\Omega{}^\#)\}$ $\Omega^\# \subset \Omega$ (for example, when $\Omega^\#$ the behavior of an *incomplete* system (Mesarovic, 1975)), because the case in point here is an exogenic representation of the subsystem (the endogenic characterization of the partial behaviour of the *D*-system will be given below, in Definition 4); in this connection, it is worth noting one attractive (if not surprising) fact that all D-systems are subsystems of a *linear* (!) *D*-system $\{(T, S_T),(W,S_W),(W^T,S_w{}^T)\}$.

It is obvious that the construction of different systems that are immanent to .D-systems opens up limitless possibilities for experimentation (see, for example, Ch. 1 (Matrosov, 1980)). Yet the problem of studying arbitrary systems is too general to be of real value, because the higher is the level of abstraction, the larger is the amount of information about details that is lost. For this reason, it is on details (primarily the "division" of the structure of partial behavior, $S_\Omega{}^\#$ that our attention will be focussed in the subsequent discussion).

Let $(X, \|\bullet\|_X)$ be a real finite-dimensional Banach space, and let $t_0 < t_1$, $T = [t_o,t_1]$ be a segment of a numerical straight line R with the Lebesgue measure μ. Let $L_P(T,\mu,X)$ $\{I < p < \infty)$ designate the Banach space of equivalency classes (mod μ) of all μ-measurable mappings $\psi: T \to X$ which are summed in the sense of Bochner and by the norm

$$\| \psi \|^{X}_{\mu,p} = \Big(\int_T \|\psi(t)\|_X^p\, \mu(dt)\Big)^{1/p},$$

through the H-space $L_p(T,\mu,R^n) \times L_P(T,\mu,R^m)$ with the norm

$$\|(\omega_1,\omega_2)\|_H = \left[\left(\|\omega_1\|_{\mu,p}^{R^n}\right)^p + \left(\|\omega_2\|_{\mu,p}^{R^m}\right)^p\right]^{1/p},$$

$\omega_1 \in L_p(T,\mu,R^n)$, $\omega_2 \in L_p\{T,\mu,R^m)$. As usual, $C(T,X)$ is the space of all continuous on T functions with the values in X and the sup-norm. Finally, $AC(T,X)$ is a linear manifold of all absolutely continuous functions from $C(T,X)$ and $\Pi = AC\{T,R^n) \times L_P(T,\mu,R^m)$. In this case the designation Π will be used (where necessary) without especially indicating that

a) the product Π is a subset in the space H (in this position the points from $AC(T,R^n)$ in the construction of Π are put in correspondence with equivalence classes from $L_p(T,\mu,,R^n)$), and

b) the topological structure in Π is the narrowing of the metric topology from H generated by the norm $\|\bullet\|_H$;

we make also the convention that positions a) and b) are also extended to subsets from Π.

For our consideration, we choose a class of linear control systems described by a vector-matrix differential equation

$$\dot{x}(t) = A(t)x(t) + B(t)u(t),\ t \in T, \qquad (1)$$

where $x(\cdot) \in AC\{T,R^n)$ is a solution of Caratheodory type (C-solution), $u(\cdot) \in L_P(T,\mu, R^m)$ is a control vector-function, $A(\cdot) \in L_p(T,\mu, \Lambda(R^n,R^n))$, $B(\cdot) \in L_p(T,\Lambda\mu,\Lambda(R^m,R^n))$, where $p,p' \in (l,\infty)$ are conjugate numbers $(1/p+l/p' = 1)$, and $\Lambda(R^m,R^n)$ is the Banach space (with the operator norm) of all linear operators operating from R^m into R^n; it is assumed that a nonstrict expression of the form: *"pair $(x,u) \in \Pi$ - C- solution of the system (1)" is allowed,* if (x,u) pointwise μ-almost everywhere in T satisfies equation (1) for a *certain* ($\exists$) pair of operators $(A, B) \in L_{p'}(T,\mu,\Lambda(R^n,R^n)) \times L_{p'}$(T,(,((Rm,Rn)), and the pair (A, B) itself will be referred to as the (A,B)-model of the system (1).

Following the thesis that "... the starting point of the process of creating any models is provided by observations and assumptions about the existence of an interrelationship between them ..." (Mesarovic, 1975), we introduce

Definition 4. *The partial behaviour $P \subset E$ of a linear continuous D-system $\{(T, S_T), (R^{N+M}, S_R^{n+m}), (E, S_E)\}$, $E \subset \Pi$ possesses:*

- a structure of ordinary linear-differential compatibility (or, eqivalently, a structure of OLD-compatibility) if either $P = \varnothing$, or there exists such a linear differential system (1) that P is contained in the class of its C-solutions (in this statement, P is said to be an OLD-compatible set);

- a structure of distributed linear-differential compatibility (or a structure of DLD-compatibility) of class k when either $P = \varnothing$, or any κ elements of an absolutely convex hull of the set P produce an OLD-compatible set (in this statement, P is said to be an DLD-compatible set of class k).

Remark 1. a) Clearly the existence in P of a structure of OLD- (DLD)-compatibility assumes implicitly that: S_R^{n+m} is a structure of the Banach space, b) The presence in P of a structure of OLD-compatibility guarantees no uniqueness of the *(A, B)*-model, for which the system (1) "realizes" P, whereas the presence in P of the structure of DLD-compatibility of an arbitrary class is insufficient for P to be OLD-compatible (see example 1 (Daneev, 1999b)). c) The condition of absolute convexity is essential because the position is possible when any " set" of k elements of the convex hull of the set P is OLD-compatible, while the P does not possess DLD-compatibility of class k.

The following assertion demonstrates that the structure of DLD-compatibility is a structure of finite character (Engelking, 1985), which cannot be said in relation to the structure of OLD- compatibility.

Assertion 1. *Let $N \subset \Pi$ and k be a certain (any) natural number. Then the structure of DLD-compatibility of class k with respect to N is a structure (property) of finite character.*

Following the statement that "... the realization theory for the class of dynamic systems addresses itself to the questions of existence of a dynamic representation for a properly defined time system ..." (Mesarovic, 1975), we stipulate that the problem of "analysis of types", for the class of linear continuous D-systems, is formalized by:

Definition 5. *Let $N \subset \Pi$ be such that there exists P^* (similarly, $P^\#$) a nonempty maximal (with a quasi-ordering with respect to a theoretical- multivariate inclusion) subset from N that possesses a structure of OLD-compatibility (accordingly, a structure of DLD-compatibility of class k). Then the linear space E^* (similarly, $E^\#$) that is spanned on P^* (accordingly, on $P^\#$) is said to be an ordinary stratum over N (a distributed stratum of class k over N), and if $N \subset E^*$ ($E^\#$), it will be said to be homogeneous.* [4]

Note (Daneev, 1995) that the pair $(A(\cdot),B(\cdot)) \in L_{p'}(T,\mu,\Lambda(R^n,R^n)) \times L_{P'}(T, \Lambda\{R^m,R^n))$ of the system (1) that includes in the class of its C-solutions an ordinary (homogeneous) stratum over N, is a strong (irrefutable) (A,B)-model over N. Therefore, the existence of a strong (A,B)-model reduces geometrically to the existence of an ordinary stratum (Daneev, 1999b). It is also worth noting that, by assertion 1, any $N \subset \Pi$ $(N \neq \varnothing)$ in accord with the Teichmuller-Tukey lemma (Engelking, 1985), either does not contain a (nonempty) set with the structure of DLD-compatibility, or over N there exists a distributed stratum (possibly not only one); a realization (in the sense of Erugin-Kalman-Mesarovic (Erugin, 1952; Kalman, 1969; Mesarovic, 1975)) for a distributed stratum can be constructed in terms of the system (1) (with a strong irrefutable (A,B)- model) with the observer (see Section 5).

[4] The OLD(DLD)-structure is invariant to the Span operator. In (Daneev, 2000) it showed that in a class of passive trajectories ($u(\cdot) = 0$) the ordinary and distributed strata coincide (i.e. the OLD and DLD structures are equivalent).

In (Daneev 2000) it showed that the geometry of strata is closely related to the solutions of OLD-

DLD-*expansions*, i.e. the problems that form the foundation for the *"synthesis of types "* in the class of linear continuous D-systems.

Definition 6. *Let $E_1,E_2 \subset \Pi$ - be linear manifolds possessing structures of OLD-compatibility (DLD-compatibility of class k). The algebraic OLD-expansion (accordingly, the algebraic DLD-expansion of class k) of the pair (E_1,E_2) is said to be a linear set E_1+E_2, if this is OLD-compatible (accordingly, DLD-compatible of class k), and $E_1 \neq E_1+E_2$, $\neq E_2$.*

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The goal of this paper is to discuss the issue of "bourbakization" of the structural identification problem as an analytical unified approach ensuring the "unity of style" of the five research areas listed in Section 1 (in terms of elucidating the typicalness of existence of the solution to this problem for structural criteria of OLD-DLD-compatibility-expansion). Furthermore, results derived from carrying out this study may serve as an indication of particular practical situations in which it is possible to expect (at least in principle) a solution of the structural identification problem in the class of *OLD (DLD)-compatible D-systems.* Of course, a "typical decidability" is a purely qualitative characteristic. It does not contain information about how well calculations of applied character are posed. Therefore, it is worth noting once again that the significance of results reported in this paper must be considered (and appreciated) precisely in the light of the above-mentioned possibility which emerges at least when the conditions of the above typical decidability are satisfied.

3. BASIC THEOREMS OF ANALYSIS AND SYNTHESIS OF TYPES

Let $L\{T,\mu,R)$ - be the space of equivalency classes of all μ-measurable on T real functions, and let $\underset{\mathrm{mod}\,\mu}{\leq}$ - be a quasi-ordering in $L(T,\mu,R)$, such that $\psi_1 \underset{\mathrm{mod}\,\mu}{\leq} \psi_2$ $(\psi_1,\psi_2 \in L(T,\mu,R))$, when $\psi_1(t) \leq \psi_2(t)$ is pointwise μ-almost everywhere in T; in this case the least upper bound of the subset W from $L(T,\mu,R)$ (if such one exists with respect to a partial ordering $\underset{\mathrm{mod}\,\mu}{\leq}$) will be designated as $\sup_L W$. Further, we introduce a nonlinear operator $\Phi : \Pi \to L(T,\mu,R),$ defined by the following construction (Lakeyev, 1998):

$$\Phi(x,u) = \begin{cases} \left\|\dot{x}(t)\right\|_{R^n} / \left\|(x(t),u(t)\right\|_{R^{n+m}}, \\ \qquad (x(t),u(t)) \neq 0; \\ 0, (x(t),u(t)) = 0, \end{cases} \tag{2}$$

where $\|\bullet\|_{R^n}$ and $\|\bullet\|_{R^{n+m}}$ - are arbitrary fixed norms, respectively, in R^n and R^{n+m}, and it is not assumed *a priori* that $\{(x,u)\}$ is an OLD- compatible set; by (Lakeyev, 1998), $\{t \in T : \dot{x}(t) = 0\} \supset \{t\in T: (x(t),u(t)) = 0\}$ (modμ) holds, which permits us to assert that the operator Φ does not lose (because of the second line of the system (2)) a priori information at the points of the set $\{t\in T: (x(t),u(t)) = 0\}$ about the behaviour of the *D*-system that "engendered" a dynamic process $t \to \{x(t),u(t)): T \to R^{n+m}$.

Let us now demonstrate that the construction of (2) is intrinsically based on the well-known (in variational analysis) Rayleigh-Ritz ratio (Horn, 1986). Let $(\Gamma, z) \to$ rel (Γ,z): $\Lambda\{R^{n+rn}, R^{n+m}) \times R^{n+m} \to R$ - be a given ratio, where Γ and z - are the matrix and the vector of corresponding dimensions, respectively. Then, if it is assumed that $(x(\cdot),u(\cdot))$ is a certain (any) C-solution of the system (1) with the (A,B)-model $(A(\cdot),B(\cdot))$ and the Φ-operator of (2) with the Euclidean norms $\|\bullet\|_{R^n}$ and $\|\bullet\|_{R^{n+m}}$, then in view of the system (1) we have $\Phi(x,u)(t)=$ (rel $(\Gamma,(x,u)))^{1/2}(t)$, $\Gamma(\cdot) = [A(\cdot), B(\cdot)]'[A(\cdot), B(\cdot)]$, where (') is the matrix transposition operation. Therefore, in the subsequent discussion the operator Φ will be referred to (irrespective of the form of the norms $\|\bullet\|_{R^n}$ and $\|\bullet\|_{R^{n+m}}$, involved in terms of (2) in its construction) as the *Rayleigh-Ritz operator,* the idea of further developing the analytical approach to solving the existence problem of strong *(A,B)*- models on the basis of diferentiating the measures

$$\nu = \int \left(\|x(t)\|^p_{R^n} + \|u(t)\|^p_{R^m}\right)\mu(dt)$$

and

$$\nu_- = \int \left(\left\|\dot{x}(t)\right\|_{R^n} \right) \mu(dt),$$

was suggested for the first time by these authors in the Conclusions of the paper (Daneev, 1995), and a start was made on its implementation in (Lakeyev, 1998).

Definition 7. *The Rayleigh-Ritz operator Φ is semiadditive on $E \subset \Pi$ with the weight α* (const) *if $\Phi(\omega_1 + \omega_2) \underset{\mathrm{mod}\,\mu}{\leq} \alpha\,\Phi(\omega_1) + \alpha\,\Phi(\omega_2)$ holds for any pair* $(\omega_1,\omega_2) \in E \times E$.

Lemma 1. *Let Φ^* and Φ^{**} be the Rayleigh-Ritz operators which mutually differ by the analytical representation of the norms* $\|\bullet\|_{R^n}$ and $\|\bullet\|_{R^{n+m}}$ *(from* *their constructions (2)), and let E be a subset from* Π, *on which the operator* Φ* *is semiadditive with a certain weight. Then the operator* Φ** *is also semiadditive on E with a certain weight.*

Below, when defining the construction of an absorbing set, we will follow (Yosida, 1965) without imposing any constraints on Card $N \subset \Pi$.

Theorem 1. *Let* $N \subset \Pi$ *be a fixed, nonempty set of observations* Φ *the Rayleigh-Ritz operator, and* к *an arbitrary natural number* ≤ dim Span *N, and let Q be a certain (or, equivalently, any) absorbing set in* Span *N. Then:*

- Span *N is a homogeneous ordinary stratum over N, if and only if there exists* $\varphi = \sup_L \Phi[Q] \in L_{P'}(T,\mu,R)$*;*

- Span *N is a homogeneous distributed stratum of class 1, if and only if* $\Phi[Q] \subset L_{P'}(T,\mu,R)$*;*

- *if, provided that b) holds, the operator Φ is semiadditive with a certain weight on any k-dimensional subspace from Span N, then Span N is a homogeneous distributed stratum of class k.*

It can be shown that if at least one of the conditions (necessary and sufficient) of Item a) of Theorem 1 is satisfied for $N \subset \Pi$, then there exists a denumerable set $Q^* \subset Q$, such that $t \to \varphi(t) = \sup\{\Phi(\omega)(t): \omega \in Q^*\}$ (here *Q, Φ* and are the constructions of the Theorem (1); on the other hand, Item c) of this Theorem gives a direct

Corollary 1. Let $N \subset \Pi$ *and* $E^{\#}$ *be a finite- dimensional distributed stratum of class k over N. Then* $E^{\#}$ *is an ordinary stratum over N, if the Rayleigh-Ritz operator is semiadditive on* $E^{\#}$ *with a certain weight.*

Theorem 1 (and it would be instructive to compare it with Theorems 1 and 2 (Daneev, 1995) and Theorem 1 (Daneev, 1999b)) makes it possible to formulate especially compactly (as against Theorem 3 (Daneev, 2000)) the conditions of OLD- DLD-expansions.

Theorem 2. *Let* $E_1, E_2 \subset \Pi$ *be linear manifolds, possessing structures of OLD-compatibility (structures of DLD-compatibility of class k). Then the linear manifold* $E_1 + E_2$*, is such that* $E_1 \neq E_1 + E_2 \neq E_2$ *is an algebraic OLD-expansion (algebraic DLD-expansion of class k) of the pair* (E_1, E_2)*, if the Rayleigh-Ritz operator is semiadditive on* $E_1 + E_2$ *with a certain weight.*

Remark 2. The merit of the OLD-expansion criterion that is formulated by this Theorem lies in the fact that it permits to infer the presence of the above-indicated expansion from "intrinsic" properties of the linear manifolds E_1 and E_2, where as, if we resort to the criterion from position a) of Theorem 1, it is necessary to "guess" the function $\varphi \in L_{P'}(T,\mu,R)$, for any $\omega \in (E_1 + E_2)$, $\Phi(\omega) \underset{\bmod \mu}{\leq} \varphi$.

It is obvious that every more-or-less advanced theory assumes the existence of a sufficient "number" of particular objects of its analysis. Therefore, although we now have Theorem 2 at our disposal, it is not unreasonable to point out the "immanent" objects of the Rayleigh-Ritz operator.

Lemma 2. *For the Rayleigh-Ritz operator* Φ, *the number* $\alpha \geq 0$ *and the set* $N \subset \Pi$, *such a property of* Φ, *as semiadditivity with a weight α is the property of finite character for a subset from N.*

Assertion 2. *Let there be: E, a linear set in* Π, *a Rayleigh-Ritz operator* Φ, *and* $\alpha \in [1,\infty)$. *Then there exists a maximum (in the case of an ordering with respect to a theoretical-multivariate inclusion) non-zero linear set* E_a *in E, on which the operator Φ is semiadditive with a weight α. With these assumptions, if E is an OLD-compatible set, closed in H, then* E_a *will also be such one.*

Remark 3. It can be shown that in *E* there exists a maximum set, on which Φ is semiadditive with a weight $\alpha \in [0,1)$, in this case, however, it can no longer be a linear set except for a trivial variant $E_a = \{0\} \subset \Pi$.

Corollary 2. *Let E be a linear OLD-compatible set, on which the operator* Φ *is semiadditive with the weight* $\alpha \in [1, \infty)$. *Then the closure of E in H will also possess a similar property.*

It is clear that $E_{a1} \subset E_{a2}$, as $1 \leq \alpha_1 \leq \alpha_2$. On the other hand, E_a depends on the initial set *E*, and of special interest is the case when *E* is an ordinary stratum. It is obvious that in the universe of all ordinary strata in *H* a special role is played by the family of all strata over Π. Their significance is due to the fact that, by Theorem 4 (Daneev, 1994b) and Lemma 1 (Daneev, 1995), each such stratum, representing the behaviour of a linear continuous D-system (with *a full input* (Mesarovic, 1975)), defines in a one-to-one manner the differential system (1) that realizes it. Thus the family of all ordinary strata over Π is isomorphic to the space of equivalency classes (modμ) of all (A,B)-models of the system (1). The last statement is intimately related to the "number" of linear manifolds in each such stratum, on which the Rayleigh-Ritz operator is semiadditive (with the weight of a "heavy" unity).

Corollary 3. *Let E* be a certain ordinary stratum over* Π, Φ *the Rayleigh-Ritz operator, and* $\alpha \in [1, \infty)$. *Then there exists a maximum set* E_a *in* E^*, *on which* Φ *is semiadditive with a weight a, with* E_a *being a linear manifold closed in H.*

The question that remains open is: Does there exist for any distributed stratum $E^{\#}$, closed in *H*, when $\alpha \in [1, \infty)$, a maximum linear set from this stratum, closed in *H*, on which the Rayleigh-Ritz operator is semiaddtiive with a weight *α*? An affirmative answer to this question would mean, in particular, that Corollary 3 is valid for *a nonloose* stratum over Π (a distributed stratum is *loose* if its closure in H does not possess a structure of DLD-compatibility).

4. CONCLUSION

"... in §10.13, we will give a new and (hopefully) exhausting account of the realization theory of linear

systems with a continuous time..." (Kalman, 1969, 10.0). The elements and possible avenues of inquiry into linear theoretical-model analysis that have been presented above, show that major work is *only getting under way!*

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